IP 7

Asymptotic Distortion of the Distributed Karhunen-Loève Transform

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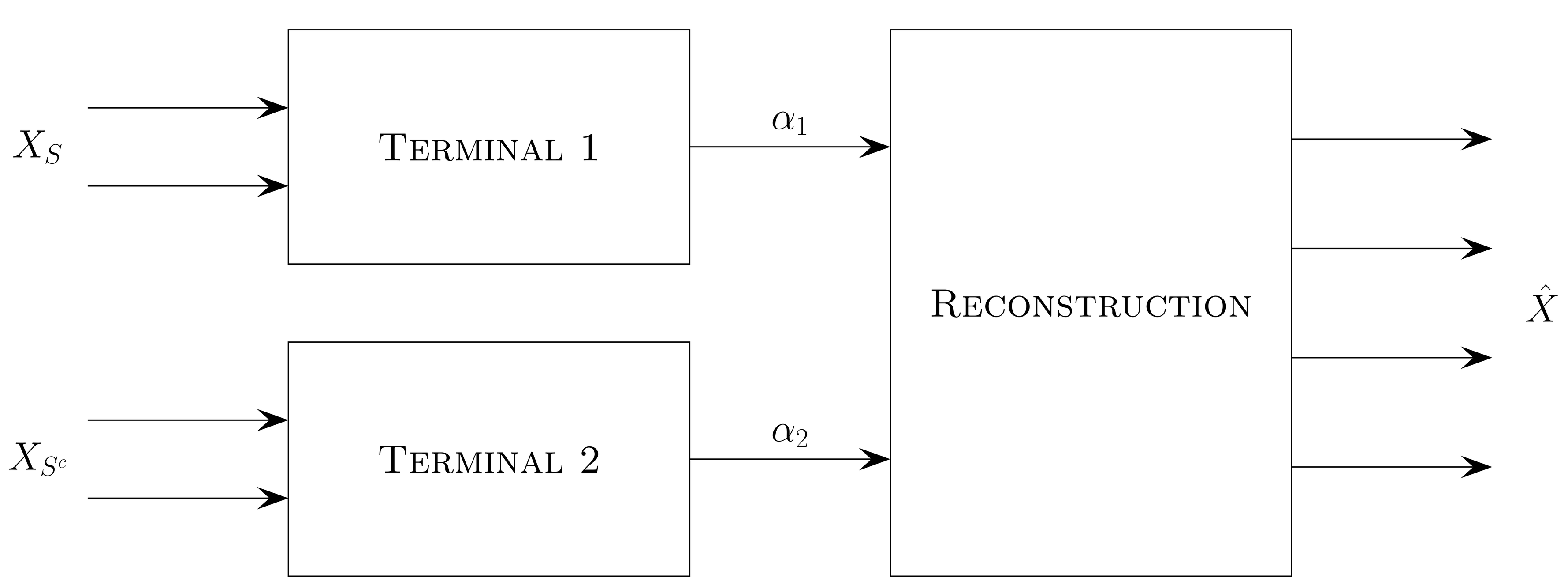
Motivation

- Signal approximation in centralized and distributed scenarios.
- Precise understanding of the reconstruction error is crucial:
  - Efficient distributed communication.
  - Robustness to sensor failure.
  - Scaling behavior.
- The distributed Karhunen-Loève Transform (dKLT) [1] plays a fundamental role in this context.
- Goals:
  - Study the asymptotic distortion behavior of the dKLT.
  - Apply our results to a simple correlation model (first-order Gauss-Markov).

Problem Setup

- Distributed approximation of a jointly Gaussian random source $X^T = (X_S^T, X_{S^c}^T)$.
  - Terminal 1: a fraction α_1 of transformed coefficients to describe $X_S \in \mathbb{R}^n$.
  - Terminal 2: a fraction α_2 of transformed coefficients to describe $X_{S^c} \in \mathbb{R}^n$.
- We want to minimize the mean squared reconstruction error $\mathrm{E}[\|X - \hat{X}\|^2]$.



Block diagram of the dKLT with two terminals.

- The general problem is difficult:
  - Globally optimal transform unknown to date.
  - Locally optimal solution: dKLT algorithm [1].
  - Distortion not tractable analytically.
- Analysis of the mean squared reconstruction error using the theory of large Toeplitz matrices: asymptotic normalized distortion.
- We study the following cases:
  - Joint processing (terminal 1 and 2 are merged): usual KLT with $\alpha = (\alpha_1+\alpha_2)/2$.
  - Distributed processing:
    - $\alpha_1 = 1$ or $\alpha_2 = 1$: conditional KLT (cKLT).
    - $\alpha_1 = 0$ or $\alpha_2 = 0$: partial KLT (pKLT).

Asymptotic Normalized Distortion

- Joint processing: the asymptotic normalized distortion for a stationary process with power spectrum density $f(\lambda)$ is derived as

$$D(\alpha) = \frac{1}{2\pi} \int_{\lambda: f(\lambda) \leq x_\alpha} f(\lambda)\, d\lambda$$

  where $F(x)$ denotes the corresponding limiting eigenvalue distribution and x_α satisfies $F(x_\alpha) = 1 - \alpha$.
- Distributed processing: extension to $D(\alpha_1, \alpha_2)$. We provide the corresponding formulas for the cKLT ($D(\alpha_1, 1)$) and the pKLT ($D(\alpha_1, 0)$).
- Exact characterization of the gain/loss due to side information/hidden part can thus be provided.

Example: first-order Gauss-Markov

- The process is obtained as follows:

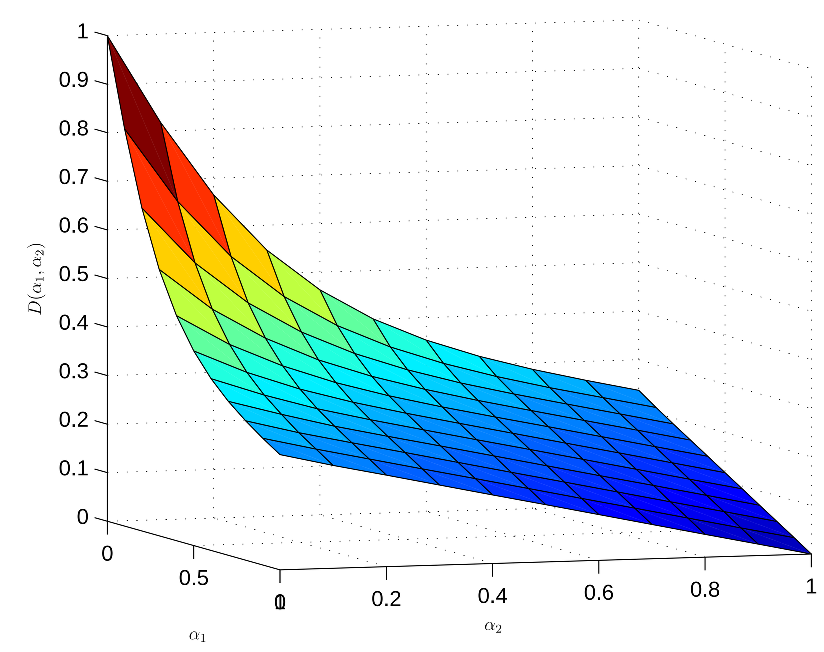
$$X_n = \rho X_{n-1} + Z_n$$

  with correlation parameter $\rho \in (0,1)$ and Z_n i.i.d. zero mean Gaussian random variable with variance $1 - \rho^2$.
- We consider the following separation: odd samples (X_S) and even samples (X_{S^c}).
- Joint processing:
  - Closed-form formulas for the asymptotic normalized distortion (left plot) for $\rho = 0.1, 0.2, \ldots, 0.9$ (top to bottom).
  - Finite dimensional approximation (right plot) for $n = 10$ and $\rho = 0.5$.

- Distributed processing:
  - Closed-form formulas for the asymptotic normalized distortion of the cKLT (left plot) and the pKLT (right plot) for $\rho = 0.1, 0.2, \ldots, 0.9$ (top to bottom).

  - We precisely quantify the gain/loss due to side information (left plot) and hidden part (right plot).

  - The above results correspond to the borders of the general asymptotic normalized distortion surface for arbitrary α_1 and α_2 shown below for $\rho = 0.6$ (numerical experiment).



References

[1] M. Gastpar, P. L. Dragotti and M. Vetterli, "The distributed Karhunen-Loève transform", submitted to IEEE Transactions on Information Theory, November 2004.

[2] O. Roy and M. Vetterli, "On the asymptotic distortion behavior of the distributed Karhunen-Loève transform", to appear in the Forty-Third Annual Allerton Conference on Communication, Control and Computing, September 2005.

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